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**GEORGE C. MARSHALL**

**SPACE  
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**HUNTSVILLE, ALABAMA**

ROOT-MEAN-SQUARE ERROR ANALYSIS FOR EQUATIONS IN  
RAWINSONDE EVALUATION PROGRAM

OTS PRICE

By

Bettye Anne Case

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By Bettye Anne Case

ABSTRACT

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An error investigation is made for the rawinsonde evaluation program presented in MTP-AERO-62-41. Standard deviations and root-mean-square errors are considered synonymous, and the errors are considered to be normally distributed.

This paper assumes rms errors for each of the five basic rawinsonde-measured atmospheric parameters, determined on the basis of available previous studies. The manner in which these errors propagate errors in the evaluation equations is examined. A computer procedure is used to approximate the partial derivatives necessary, utilizing the original evaluation program.

Selected thermodynamic and wind quantities computed from rawinsondes released near launch time for Saturn SA-1 and Saturn SA-2 are presented graphically with their corresponding rms errors, and a brief discussion is given of the variation between angle-of-attack and rawinsonde-measured wind evaluations.

Within the limits of the accuracy of the assumed errors in the basic variables and the necessity for computer approximations, rms error values computed by the procedures in this paper give an indication of the nature and magnitude of errors in rawinsonde evaluations.



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TERRESTRIAL ENVIRONMENT SECTION  
AEROPHYSICS AND ASTROPHYSICS BRANCH  
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## DEFINITION OF SYMBOLS\*

Symbol	Definition
$f(x)$	Function dependent on one variable $x$
$F(x, y, \dots)$	Function dependent on one or more variables $x, y, \dots$
$\Delta x$	Change (error or increment) in the variable $x$
$\Delta f$	Change (error or increment) in the value of $f(x)$
$\sigma_x$	Standard deviation (root-mean-square error) of the variable $x$
$\sigma_f$	Standard deviation (root-mean-square error) of the function $F(x, y, \dots)$
$\frac{df}{dx}$	Derivative of $f(x)$ with respect to $x$
$F_x$	Partial derivative of $F(x, y, \dots)$ (with respect only to $x$ )
$\theta$	Elevation angle (between plane tangent to earth and line of sight) (degrees)
$\psi$	Azimuth angle (between projection of line of sight in plane tangent to earth and true north) (degrees)
$p$	Pressure (mb)
$r_a$	Relative humidity (%)
rms	Root-mean-square (error); used synonymously with standard deviation, the necessary assumptions for such usage being made
$r_{xy}$	Coefficient of correlation

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\*Symbols and subscripts used only in Tables IV and V are defined therein, using same notation as Ref. 3.

## DEFINITION OF SYMBOLS (CONT.)

Symbol	Definition
t	Temperature (°C)
~	Is approximately equal to

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ROOT-MEAN-SQUARE ERROR ANALYSIS FOR EQUATIONS IN  
RAWINSONDE EVALUATION PROGRAM

by Bettye Anne Case

SUMMARY

An error investigation is made for the rawinsonde evaluation program in current use at Marshall Space Flight Center (Ref. 3). The errors are considered to be normally distributed; standard deviations and root-mean-square errors are considered synonymous.

This paper assumes rms errors for each of the five basic rawinsonde-measured atmospheric parameters on the basis of studies. The manner in which these errors propagate errors in the evaluation equations is examined. A computer procedure is used to approximate the partial derivatives necessary in the error determination. This procedure utilizes the original evaluation program.

Values in the original evaluation program are based on smoothed position coordinates. Therefore, some of the small-scale variations occurring in unsmoothed rms errors are eliminated through use of five-point running means.

Selected thermodynamic and wind quantities computed from rawinsonde measurements made in support of Saturn SA-1 and Saturn SA-2 flight tests are presented graphically with corresponding rms errors, together with a brief discussion of the variation between angle-of-attack and rawinsonde-measured wind evaluations.

Within the limits of the assumed errors in the basic variables and the necessity for computer approximations, rms error values are computed by the procedures in this paper. The results give an indication of the nature and magnitude of errors in rawinsonde evaluations.

## SECTION I. INTRODUCTION

Rawinsonde measured atmospheric parameters are currently evaluated at Marshall Space Flight Center as described in Ref. 3. It is desired to provide those using these evaluations some information on the amount and nature of error in these evaluations for proper consideration in analyses involving meteorological considerations. This is of particular importance for rawinsonde values used in flight evaluation of a Saturn or other major space vehicle.

There are many previous studies which deal with the errors in quantities computed from rawinsonde measurements. (Some of these are Refs. 1, 2, 4, 5, 7, 8, 9, 10, 11, 12, and 17.) The errors of these quantities are directly dependent on the methods used in evaluation of the rawinsonde data. This paper describes a computer procedure to determine standard deviations of functions dependent on rawinsonde measurement. This procedure will yield, at any given time or interpolated altitude point, a root-mean-square error value for any quantity which is determined by the rawinsonde evaluation program in current use at Marshall Space Flight Center.\*

The assumption that rawinsonde errors are normally distributed is generally considered valid (Refs. 1 and 12) and must be made for the purposes of this paper. It may then be considered that about 68.27% of the time the real value of a quantity will not differ from the computed quantity by more than plus or minus the corresponding standard deviation, where standard deviation and root-mean-square error are considered synonymous.

Discussion of related literature is made in this paper at the points where it is applicable, particularly in the determination of rms values for the basic rawinsonde measured atmospheric parameters. A certain amount of subjectivity was inherent in this determination. Choice of increments for use in computer approximation of partial derivatives also involved subjective decision. W. W. Vaughan, O. E. Smith, and C. C. Dalton provided the knowledge of and experience with the nature of the data necessary for these decisions.

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\*This paper is concerned with the treatment of the random errors in atmospheric measurement and not with bias errors.

## SECTION II. THEORY OF ROOT-MEAN-SQUARE ERROR PROPAGATION

When  $f(x)$  is some function of the variable  $x$ , the first term in Taylor's series is often sufficient to express the effect on  $f(x)$  of a small error  $\Delta x$  in  $x$ :

$$\Delta f \sim \frac{df}{dx} \Delta x \quad (1)$$

The assumption that the error in  $f(x)$  varies directly as the error in  $x$ , with the derivative of the function with respect to  $x$  as the constant of variation, is close enough for use when  $\Delta x$  is sufficiently small and/or evaluation of higher derivatives of  $f(x)$  yields very small values.

For functions of several variables,  $F(x, y, \dots)$ , a multi-variable Taylor's series may be used to obtain error expressions:

$$\Delta F \sim F_x \Delta x + F_y \Delta y + \dots \quad (2)$$

In the case where  $F(x, y, \dots)$  is linear in one or more of the variables, Eq. (2) is exact with respect to that or those variables. In this paper, it is assumed that for those functions which are non-linear the second and higher order derivatives are sufficiently small that Eq. (2) is a good approximation.

When the standard deviations (root-mean-square errors) of the variables upon which a function is based are known, the preceding equation leads to a relation between these rms errors which yields the variance of the function:

$$\sigma_F^2 = (F_x \sigma_x)^2 + (F_y \sigma_y)^2 + \dots + 2(F_x F_y \sigma_x \sigma_y r_{xy} + \dots). \quad (3)$$

If the variables are independent, therefore having zero correlation coefficients, the expression for rms error obviously simplifies to:

$$\sigma_F = \left[ (F_x \sigma_x)^2 + (F_y \sigma_y)^2 + \dots \right]^{\frac{1}{2}}. \quad (4)$$

(For a more detailed discussion and derivation, see Refs. 6, 13, 15, and 16.)

### SECTION III. APPLICATION OF PROPAGATION-OF-ERROR TO COMPUTER EVALUATION OF RAWINSONDE DATA

To determine the root-mean-square error in a given equation at a given time or for a given interpolated height from rawinsonde input, an expression for this equation in terms of independent variables must be available for the application of Eq. (4) above.

In the present rawinsonde (AN/GMD-1A or B) the parameters measured are temperature, pressure, relative humidity, and azimuth and elevation angles. For statistical evaluations these parameters may be considered to be independent of each other.

Propagation-of-error in the manner of Eq. (4) may be directly applied for rms error determination of atmospheric parameters after evaluation of the necessary partial derivatives.

#### A. PREVIOUS STUDIES FOR DETERMINATION OF ROOT- MEAN-SQUARE ERRORS IN BASIC VARIABLES

1. Temperature. Temperature values at Marshall Space Flight Center and Cape Canaveral, Florida, are obtained through use of a ceramic thermister and are subject to error caused by such factors as residual lag, solar radiation, calibration, recording and human factors (Ref. 11). Studies to determine the rms error in temperature, as with all of the basic variables, are not in complete agreement. Reference 1 lists  $\sigma_t = 0.36^\circ\text{C}$  from laboratory studies, but  $\sigma_t = 1^\circ\text{C}$  from consideration of other studies which had been previously reported. Johannessen (Ref. 9) lists  $\sigma_t = 0.7^\circ\text{C}$ . The Salton Sea experimental studies (Ref. 10) found absolute errors within  $0.5^\circ\text{C}$  except for one of  $0.7^\circ\text{C}$ . The Compendium of Meteorology (Ref. 8) lists probable error at  $0.5^\circ\text{C}$ , and therefore, rms error at  $0.7^\circ\text{C}$ . For the purposes of this paper,  $\sigma_t = 0.7^\circ\text{C}$  is used.

2. Relative Humidity. The relative humidity measurement is acknowledged to be the weakest link in the rawinsonde measuring system. The currently-used element is a carbon strip. The Compendium (Ref. 8) lists for relative humidity a probable error of a constant 2.5% (rms error 3.7%) under ideal conditions where temperature is to  $-10^\circ\text{C}$ , there is a humidity range of 15 to 96%, and the element is not subjected to moisture condensations. This, however,



is based on use of the older lithium chloride element (Ref. 10). Experimental studies, probably using other than a carbon strip, also found all tested elements to yield measurements accurate within  $\pm 9\%$  of the measured value (Ref. 10).  $\sigma_{r_a} = 10\% r_a$  is used in this paper.

3. Pressure. Pressure measurements are usually obtained by means of an aneroid capsule and errors are caused by the mechanical operation of the aneroid-lever system, the commutator setting, switching and temperature compensation. Use of a hypsometer at altitudes  $> 20$  km is said to yield an error of 1% of the measured pressure value (Ref. 11). The rawinsonde used at Cape Canaveral employs aneroid pressure elements with hypsometer used at higher altitudes. Johannessen gives aneroid-measured pressure errors at mathematically defined heights (Ref. 9). Using the "Reference Atmosphere for Patrick AFB" (Ref. 14) a table of pressure errors for aneroid measurements was decided upon, depending on the pressure at a given reading. (Johannessen's altitudes in feet were converted to meters and the pressure corresponding to that altitude at Patrick AFB was supplied. The results follow as Table I.)

Table A

## Assumed rms Errors in Pressure

<u>Pressure Range</u> (mb)	<u><math>\sigma_p</math></u> (mb)
$852 < p$	0.10
$588 < p \leq 852$	0.70
$395 < p \leq 588$	1.00
$260 < p \leq 395$	1.20
$158 < p \leq 260$	1.00
$95 < p \leq 158$	0.70
$57 < p \leq 95$	0.55
$30 < p \leq 57$	0.40
$22 < p \leq 30$	0.30
$14 < p \leq 22$	0.20
$p \leq 14$	0.12

4. Azimuth and Elevation Angles. The precision of angle measurement of the radio direction finder (AN/GMD-1A or B) must take into account random errors due to the swinging of the rawinsonde, dynamic errors in angle measurement at the ground, and errors present in the angle recorder. The Compendium (Ref. 8) lists probable error as  $0.05^\circ$  C. Johannessen (Ref. 9) has used this same figure for rms error, as did Vaughan (Ref. 17). This paper also uses  $\sigma_\theta, \sigma_\psi = 0.05^\circ$  or 0.000873 radians.

## B. COMPUTER APPROXIMATION OF PARTIAL DERIVATIVES

To actually compute the partial derivatives, each parameter's equation must be broken down to expressions in the five basic variables which are independent. This, however, is so laborious and leads to equations of such length that the probability of introduction of human error becomes near certainty. Computer evaluation is, therefore, utilized for approximation of the partial derivatives of each parameter with respect to each of the variables.

For a given parameter,  $F(t, r_a, p, \theta, \psi)$ , the partial derivative with respect to temperature at a specific temperature,  $t$ , may be approximated:

$$F_t \sim \frac{F(t + \Delta t, r_a, p, \theta, \psi) - F(t - \Delta t, r_a, p, \theta, \psi)}{2\Delta t} . \quad (5)$$

In like manner, by choosing  $\Delta p, \Delta r_a, \Delta \theta$ , and  $\Delta \psi$  wisely, all partial derivatives of a given equation may be approximated for each required point (Ref. 15).

A different set of increments on the basic variables should be chosen for each parameter's equation. The choice of each increment should be based on how the equation is affected by changes in the corresponding variable. This was not considered a feasible procedure in programming in view of the great number of equations in the evaluation report (Ref. 3) and the computer time which would be required for trial sets of increments. Further modification of this procedure will include attempts to determine increments more wisely. One set of increments was subjectively chosen for use with all equations on the basis of its apparent superiority over several other sets of increments tried. The set of increments currently used is:

$$\Delta t = 0.05^\circ \quad \Delta p = 0.5 \text{ mb} \quad \Delta r_a = 1\frac{1}{2}\% \quad \Delta\theta, \Delta\psi = 0.000873 \text{ radians}$$

The computer program is so set up that the original evaluation program (Ref. 3) computes the partial derivatives of every function in it with respect to each variable upon which that function depends.

#### C. COMPUTATION AND ADJUSTMENT OF ROOT-MEAN-SQUARE ERRORS

Equation (4) may now be utilized to give  $\sigma_F$  for any desired function in the original evaluation program, either time- or altitude-sequenced. The computer will have stored the partial derivatives from Eq. (5) and the assumed rms values of basic variables as described in Section III. A.

The intricate procedure used for smoothing position coordinates in the original evaluation program (Ref. 3), consequently affecting all equations dependent on position coordinates (all wind quantities), makes it impractical to present the propagated root-mean-square error exactly in relation to the smoothed values. It is assumed that the magnitude of these errors is about the same as those based on unsmoothed position coordinates. Such rms values, computed originally, appeared to have too much small-scale variation; simple five-point running means were taken to smooth out some of this small-scale variation.

#### D. TIME- AND HEIGHT-SEQUENCED DISPLAY OF ROOT-MEAN-SQUARE ERRORS

The current computer display of rms errors does not include all parameters from the original evaluation program (Ref. 3). The following parameters presented in tabular form were selected for three reasons, any one or all of which might be applicable: 1) previous studies concerning these might lend some subjective idea of reliability, 2) they have potential use for publications by the Aerophysics and Astrophysics Branch, and 3) they are used in flight evaluation by other MSFC Organizations. The tables of the computer print-out begin with Table IV; Nos. I, II, and III are used for the tables of the original evaluation program (Ref. 3).

Table IV\*

## Computed rms Error of Rawinsonde Data

<u>Column</u>	<u>Heading</u>	<u>Explanation</u>
1	TIME	Time (min) of rawinsonde flight from launch point
2	HN	rms error in PHI, geopotential height (geopotential meters), of Table I-A, Column 6
3	DPHI	rms error in layer thickness (m)
4	YS	rms error in geometric height (m) of Table I-B, Column 2
5	XS	rms error in unsmoothed spherical zonal position coordinate (m) corresponding to Table I-B, Column 10
6	ZS	rms error in unsmoothed spherical meridional coordinate (m) corresponding to Table I-B, Column 11
7	TSTAR	rms error in virtual temperature ( $^{\circ}$ K)
8	ES	rms error in saturation vapor pressure (mb)
9	EA	rms error in actual vapor pressure (mb)

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\*When Tables I, II, and III are referred to in this and the following tables, reference is being made to Ref. 3.

Table V-A

## Computed rms Error in Rawinsonde Data

<u>Column</u>	<u>Heading</u>	<u>Explanation</u>
1	Y	Spherical coordinate referenced height (m) at 250 m intervals above sea level
2	WS	rms error in scalar wind velocity (m/sec) of Table II-B, Column 2
3	TSTAR	rms error in virtual temperature ( $^{\circ}$ K) as computed in Ref. 3, page 5
4	RHO	rms error in density ( $\text{kp sec}^2 \text{ m}^{-4}$ )
5	RHOT	rms error in density ( $\text{kg/m}^3$ ) of Table I-A, Column 4
6	US	rms error in zonal wind component (m/sec) of Table II-A, Column 7
7	VS	rms error in meridional wind component (m/sec) of Table II-A, Column 8
8	NE	rms error in electromagnetic index of refraction of Table I-B, Column 9
9	WXS	rms error in pitch wind component (m/sec) of Table II-B, Column 4
10	WZS	rms error in yaw wind component (m/sec) of Table II-B, Column 5
11	SIW	rms error in wind direction (deg) of Table II-B, Column 3

Table V-B

## Computed rms Error of Rawinsonde Wind Data

<u>Column</u>	<u>Heading</u>	<u>Explanation</u>
1	Y	Spherical coordinate referenced height at 250 meter intervals above sea level
2	S(250)	rms error in wind shear over 250 meter layer of Table II-B, Column 6
3	SU(250)	rms error in zonal component of wind shear over 250 meter layer of Table II-B, Column 9
4	SV(250)	rms error in meridional component of wind shear over 250 meter layer of Table II-B, Column 10
5	SX(250)	rms error in pitch shear component over 250 meter layer of Table II-B, Column 7
6	SZ(250)	rms error in yaw shear component over 250 meter layer of Table II-B, Column 8
7	S(500)	rms error in wind shear over 500 meter layer of Table II-C, Column 2
8	SU(500)	rms error in zonal component of wind shear over 500 meter layer of Table II-C, Column 3
9	SV(500)	rms error in meridional component of wind shear over 500 meter layer of Table II-C, Column 4
10	SX(500)	rms error in pitch shear component over 500 meter layer of Table II-E, Column 3
11	SZ(500)	rms error in yaw shear component over 500 meter layer of Table II-E, Column 4

Table V-C

Computed rms Error of Rawinsonde Wind Data

<u>Column</u>	<u>Heading</u>	<u>Explanation</u>
1	Y	Spherical coordinate referenced height at 250 m intervals above sea level
2	S (1000)	rms error in wind shear ( $\text{sec}^{-1}$ ) over 1000 m layer of Table II-C, Column 5
3	SU (1000)	rms error in zonal component of wind shear ( $\text{sec}^{-1}$ ) over 1000 m layer of Table II-C, Column 6
4	SV (1000)	rms error in meridional component of wind shear ( $\text{sec}^{-1}$ ) over 1000 m layer of Table II-C, Column 7
5	SX (1000)	rms error in pitch shear component ( $\text{sec}^{-1}$ ) over 1000 m layer of Table II-E, Column 6
6	SZ (1000)	rms error in yaw shear component ( $\text{sec}^{-1}$ ) over 1000 m layer of Table II-E, Column 7
7	S (4000)	rms error in wind shear ( $\text{sec}^{-1}$ ) over 4000 m layer of Table II-D, Column 5
8	SU (4000)	rms error in zonal component of wind shear ( $\text{sec}^{-1}$ ) over 4000 m layer of Table II-D, Column 6
9	SV (4000)	rms error in meridional component of wind shear ( $\text{sec}^{-1}$ ) over 4000 m layer of Table II-D, Column 7
10	SX (4000)	rms error in pitch shear component ( $\text{sec}^{-1}$ ) over 4000 m layer of Table II-F, Column 6
11	SZ (4000)	rms error in yaw shear component ( $\text{sec}^{-1}$ ) over 4000 m layer of Table II-F, Column 7

#### SECTION IV. ERRORS IN RAWINSONDE EVALUATIONS AT LAUNCH TIME FOR SA-1 AND SA-2

The root-mean-square errors for temperature, pressure, humidity, and azimuth and elevation angles were each assumed in the manner described in Section III. From these assumed errors, and from the nature of the functions under consideration, root-mean-square errors have been approximated for the functions from Ref. 3 as indicated in Tables IV and V. This section will include several different types of graphic presentation of some of the computed quantities, selected because the nature of their errors has been previously studied, or because they are representative of those necessary to flight evaluation studies of space vehicles such as the Saturn.

The headings of the figures describe a rawinsonde release by its date, place, and time Greenwich referenced. To simplify the following paragraphs, and to indicate the connection of a given rawinsonde release time to a vehicle flight, reference will be made as follows: October 27, 1961, 1513Z, Cape Canaveral - "SA-1 t-0"; April 25, 1962, 1408Z, Cape Canaveral - "SA-2 t-0"; April 25, 1962, 1611Z, Cape Canaveral - "SA-2 t+2".

##### A. ROOT-MEAN-SQUARE ERRORS OF REPRESENTATIVE THERMODYNAMIC AND WIND QUANTITIES

The magnitude of saturation vapor pressure and of saturation vapor pressure rms error both fall with increasing time (height), as Fig. 1 illustrates, with SA-1 t-0 and SA-2 t-0 releases almost identical. During the first 33 min (10 km for SA-1 t-0 and SA-2 t-0) of the rawinsonde ascent when these values are large enough to be represented on the graph, the proportion of rms error to the evaluated parameter is increasing, at first in a constant manner, and then more rapidly, from 4.21% to 7.45%.

Graphic presentation of actual vapor pressure is difficult because of its direct dependence on relative humidity. An attempt is made, however, in Fig. 2 to show  $\sigma_{E_a}$  versus  $E_a$ . Again, a close relationship is noted between SA-1 and SA-2 releases, and the relative error remains reasonably small.



The rms error in virtual temperature receives its largest contribution through propagation of error from the assumed error in temperature. The percent of rms error to the evaluated parameter ranges from 0.25% to 0.30%. With an assumed error of  $0.7^{\circ}\text{C}$  in temperature, values for rms error in virtual temperature range from 0.7 to  $0.9^{\circ}\text{C}$ .

Figure 3 presents rms error in density plotted against density values for SA-1 t-0 and SA-2 t+2 rawinsonde releases. A high correlation again exists between this time-sequenced data. Higher correlation is shown in Fig. 4, however, which presents height-sequenced rms error in density, approaching zero with increasing altitude. The magnitude of rms error in relation to magnitude of density is small, from 0.1 to 1.3%.

Figure 5 presents several curves reflecting three times the rms error in geometric height plotted against corresponding time-sequenced geometric heights. Curve (1) reflects the relationship as derived from the SA-1 t-0, SA-2 t-0, and SA-2 t+2 releases. There is not sufficient difference between these to show graphically. Curve (2) reflects three times rms error plotted against geometric height, where the error is the contribution from pressure only, neglecting temperature and humidity. This is adapted from Johannsen's paper (Ref. 9) through the Patrick Reference Atmosphere (Ref. 14). Curve (3) is an adaption of three times rms height error versus pressure from Ref. 1, by pressure-height conversion through the Patrick Reference Atmosphere (Ref. 14). The similarity of trend between the derived curve (1) and the AWS curve (3) is apparent, the maximum error being larger for the AWS curve.

All rms errors pertaining to wind were multiplied by three so that they may be more easily read when graphed to the same scale as the quantity to which they apply. All are height-sequenced. Figure 6 presents wind speed and its  $3\sigma$  values for SA-1 t-0. Interpretation indicates reasonably accurate evaluation up to 15 km height, and doubtful evaluation as wind speeds decrease between 15 and 26 km, and, relative to the increasing wind speeds above 26 km, more accurate evaluation. Error analysis for wind components leads to conclusions comparable to those for vector wind speeds.

Figures 7 through 12 present  $3\sigma$  in various wind shear values, plotted to the same scale as the corresponding shear quantity. Somewhat

the same conclusions regarding the relative accuracy of evaluation by height levels which apply to wind speed also apply to the wind-speed-derived wind shears, except that above 26 km the relative accuracy does not improve so much.

Figures 7, 8, and 9 present wind shear values computed over 250 meter levels and attributed to the top of the level. Since both positive and negative range and crossrange component shears occur in Figs. 8, 9, 11, and 12, the graphing of error to the same scale is simply for convenience, and must be applied to the positive or the negative component as appropriate. The great amount of variation at close levels also causes difficulty in graphic presentation of these data. Figures 10, 11, and 12, presenting wind shear values computed over 1000 meter levels, are easier to read since they have less small-scale variation.

#### B. DIFFERENCES BETWEEN ANGLE-OF-ATTACK AND RAWINSONDE MEASURED WIND DATA

Close correlation is seen in Figs. 13 through 15 between angle-of-attack measured and rawinsonde measured wind data, even though differences, diverging with increasing altitude, in time and position occur. These figures are shown in this report only for intuitive comparison between the two systems of measurement. Agreement for the larger scale wind shears in both amplitude and phase between the two systems of measurement may be considered good. Statistical comparison is difficult due to the non-normality of wind distributions and other factors. A future Internal Note in preparation by O. E. Smith will discuss these problems in some detail (Ref. 18).

## SECTION V. CONCLUSIONS AND RECOMMENDATIONS

To the extent that the assumed rms errors in the basic variables are correct, and considering the inaccuracies arising through computer approximations, the rms error values computed by these procedures give an indication of the nature and magnitude of errors in the meteorological parameters when they are evaluated by the methods of Ref. 3. When considering rms errors in wind quantities, it should be remembered that rawinsonde evaluations are generally accepted as representative of the "steady state" winds only; therefore, the rms errors presented in this paper are representative of the steady state wind errors for the three particular examples. This report provides the user with a technique for determining the steady state wind errors for wind determined by individual rawinsonde (AN/GMD-1 system) measurements.

Future investigation to determine a set of basic variable increments for each function (rather than using the same set for all functions) would greatly increase confidence in the approximation of the partial derivatives required for error propagation through known rms errors in dependent variables.

A study of many sets of data evaluated in this manner may be made to determine if the trends apparent from these few evaluations for certain rms errors are reliable. Further, a study of several releases where very low elevation angles were encountered and the resulting effect on rms errors will be made.

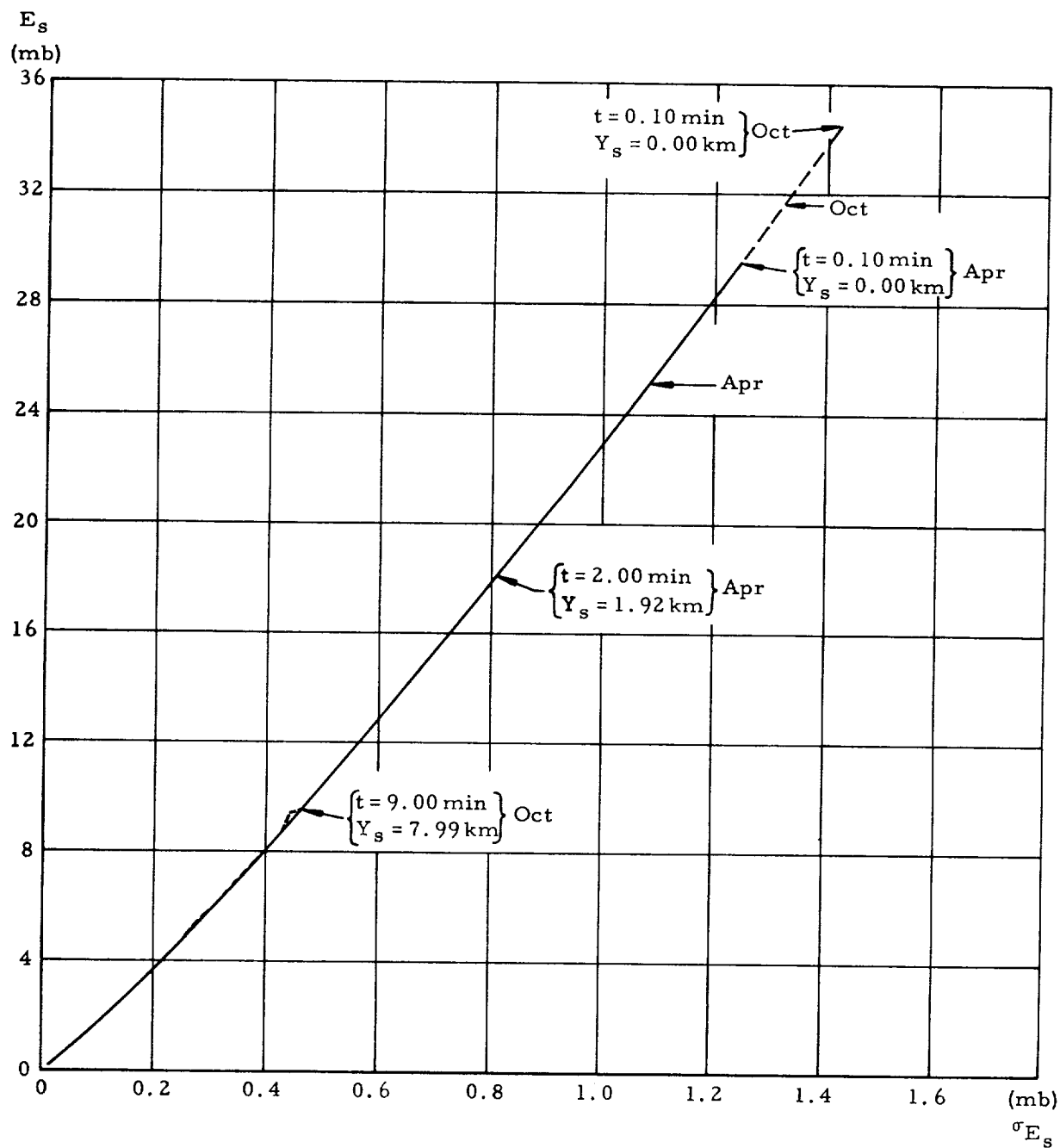
In describing the atmospheric environment of the Saturn SA-2, the method of this paper was used for error in quantities computed from rawinsonde data.

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Fig. 1. rms Error in Saturation Vapor Pressure, Cape Canaveral, Florida,  
October 27, 1961, 1513Z, and April 25, 1962, 1408Z

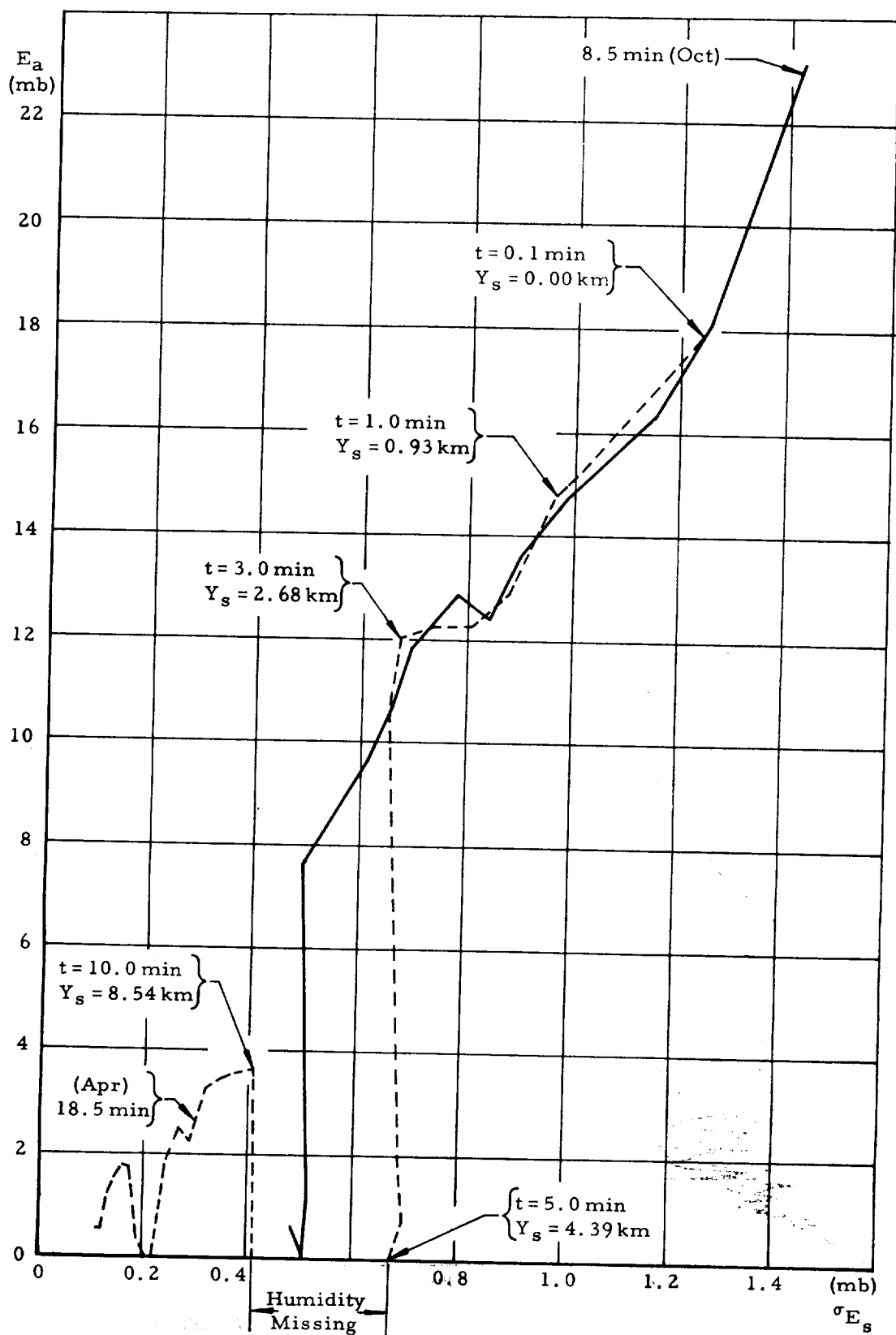


Fig. 2. rms Error in Actual Vapor Pressure, Cape Canaveral, Florida  
October 27, 1961, 1513Z, and April 25, 1962, 1408Z

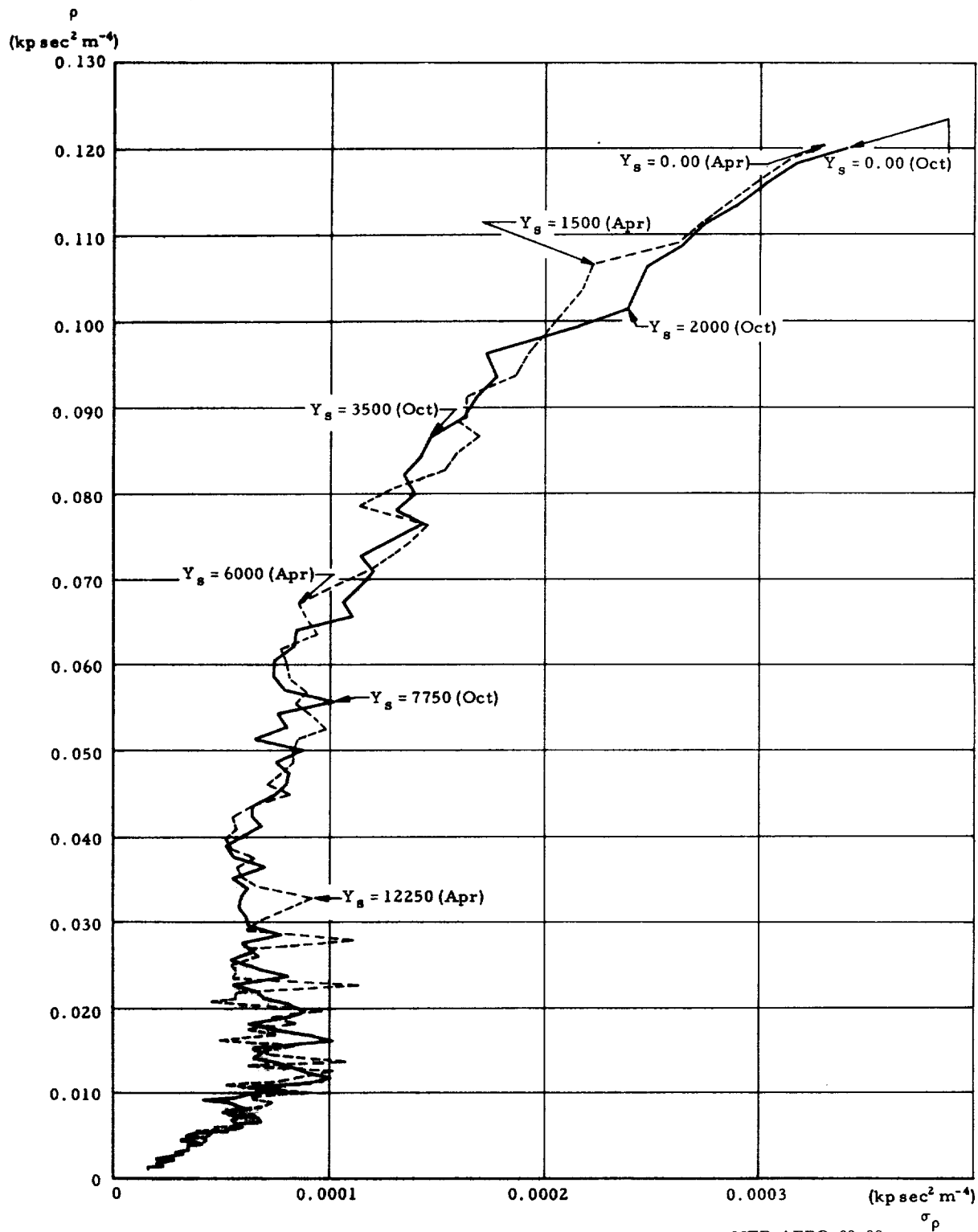


Fig. 3. rms Error in Density, Cape Canaveral, Florida,  
October 27, 1961, 1513Z, and April 25, 1962, 1611Z

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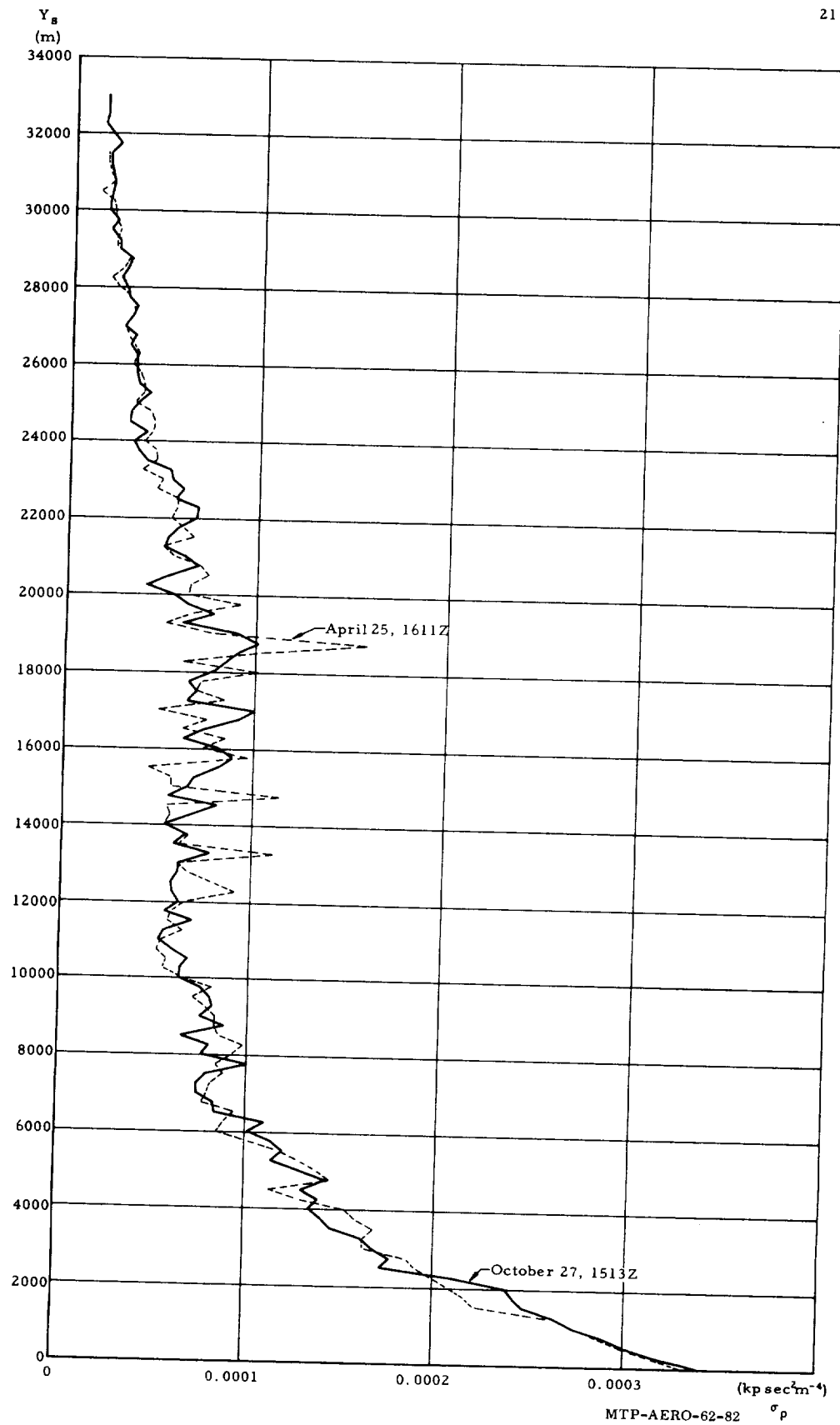


Fig. 4. rms Error in Density Versus Geometric Height,  
Cape Canaveral, Florida, October 27, 1961, 1513Z,  
and April 25, 1962, 1611Z

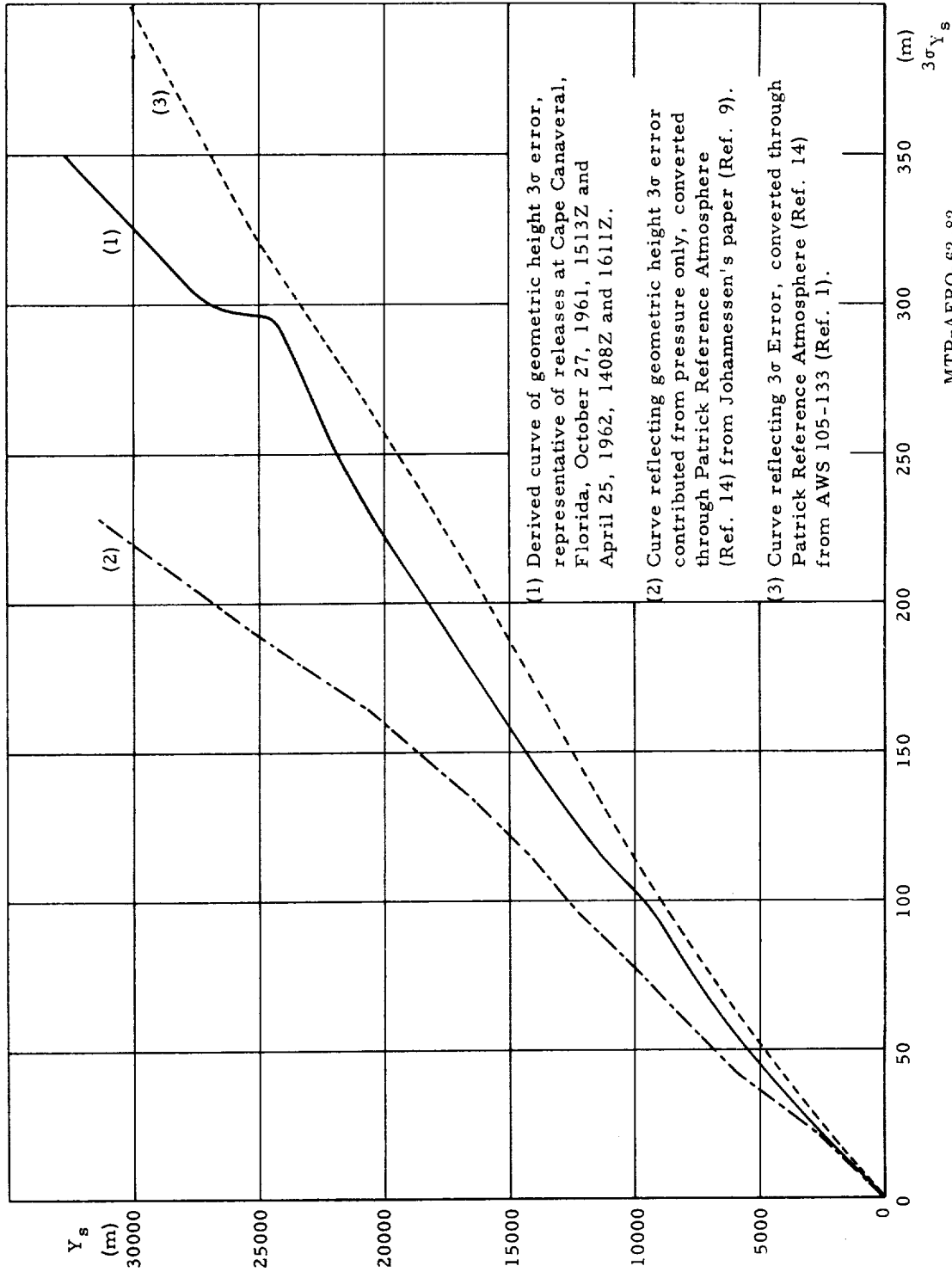


Fig. 5. Comparative Geometric Height rms Errors

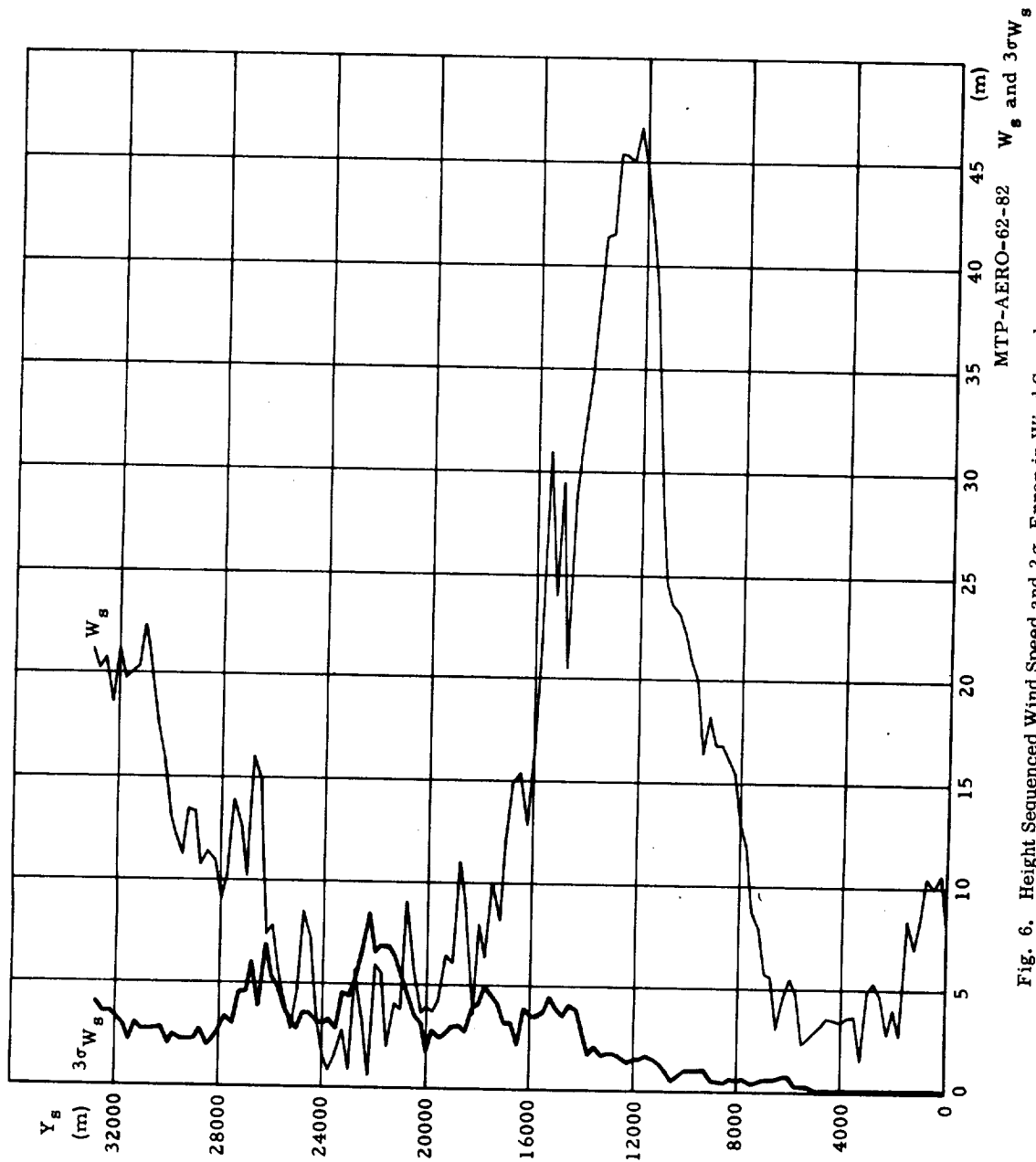


Fig. 6. Height Sequenced Wind Speed and  $3\sigma$  Error in Wind Speed,  
Cape Canaveral, Florida, October 27, 1961, 1513Z

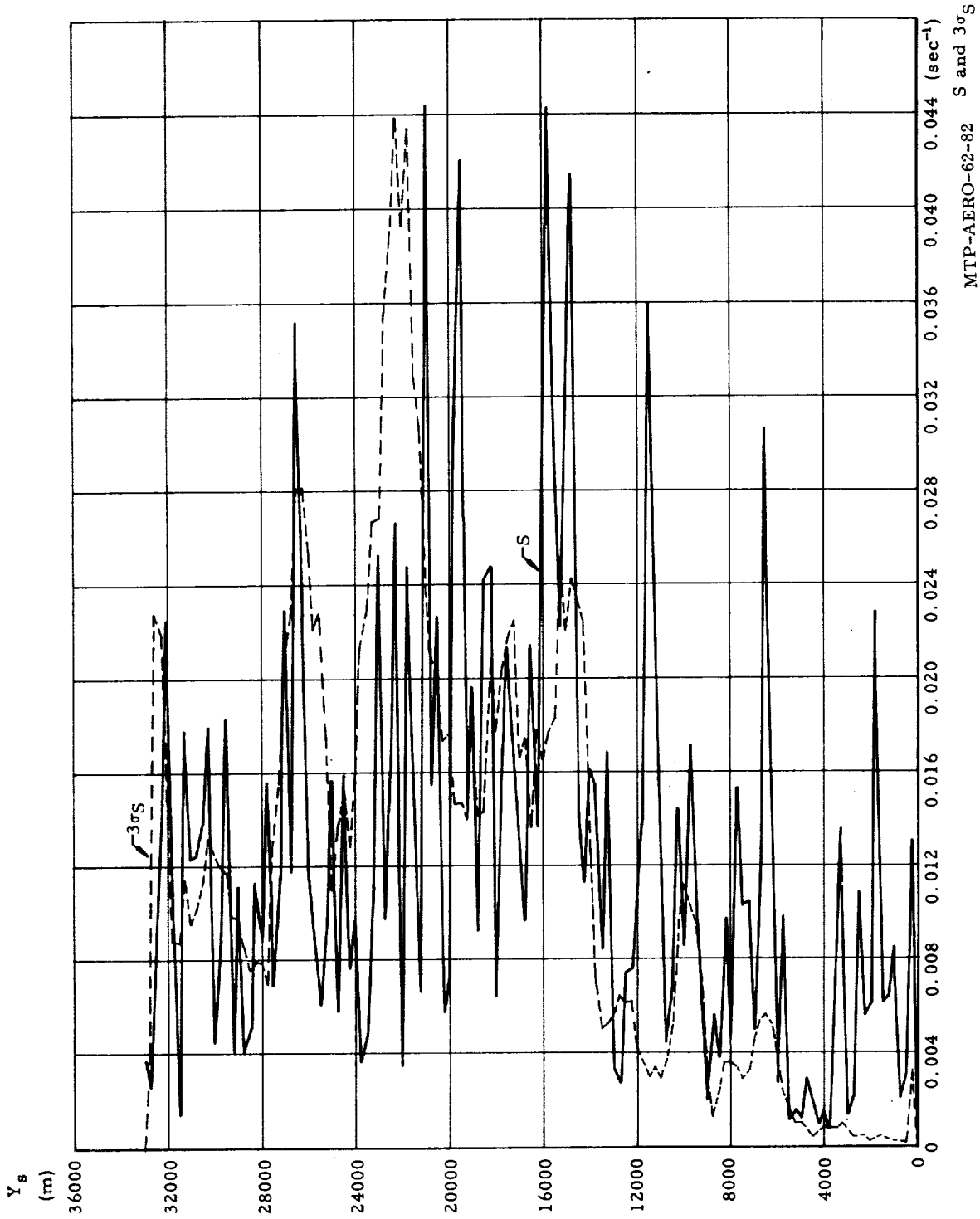


Fig. 7. Height Sequenced Wind Shear and  $3\sigma$  Error in Wind Shear over 250 m Layer,  
Cape Canaveral, Florida, October 27, 1961, 1513Z

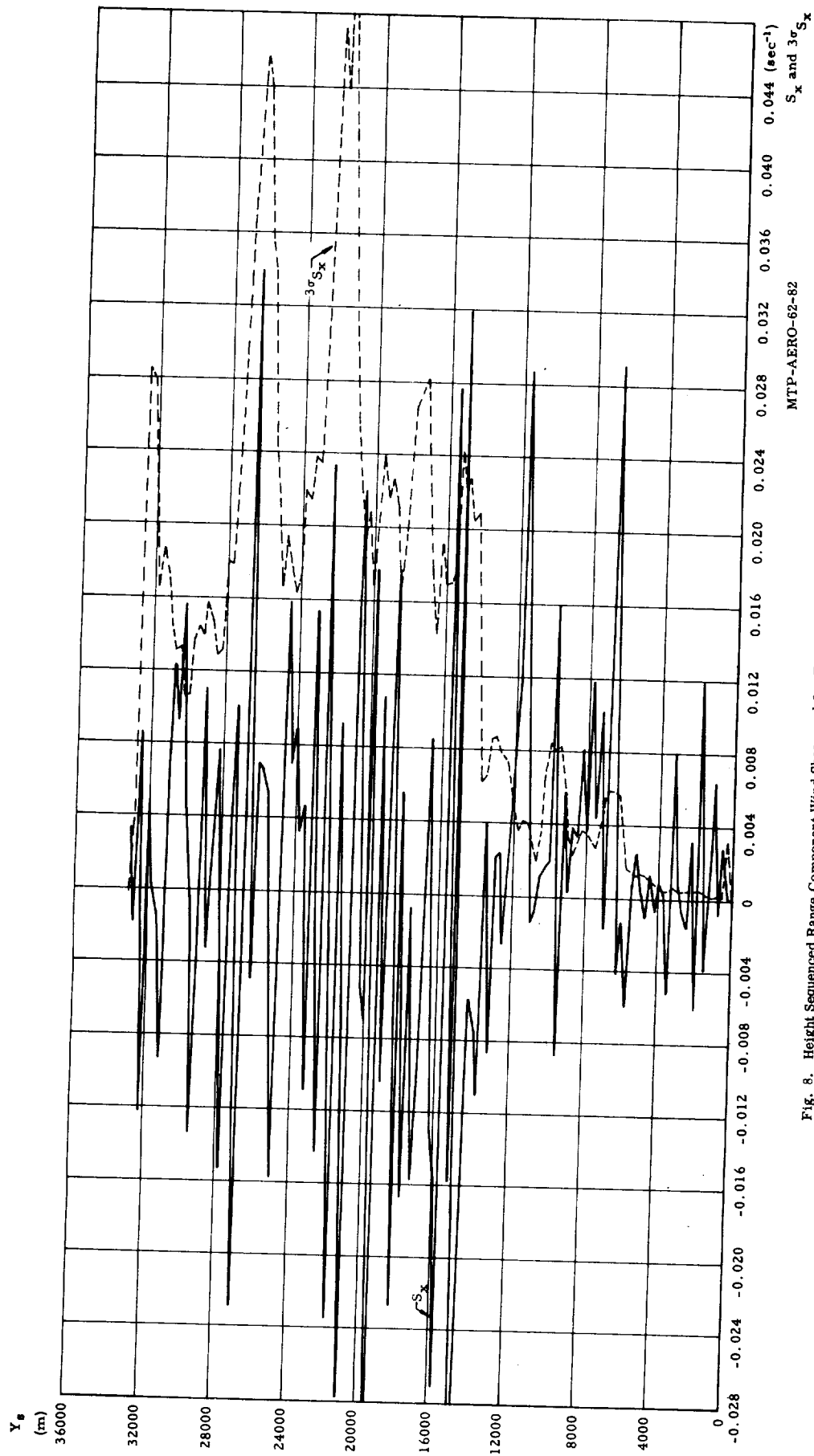


Fig. 8. Height Sequenced Range Component Wind Shear and  $3\sigma$  Error in Range Component Wind Shear, over 250 m Layers, Cape Canaveral, Florida, October 27, 1961, 1513Z

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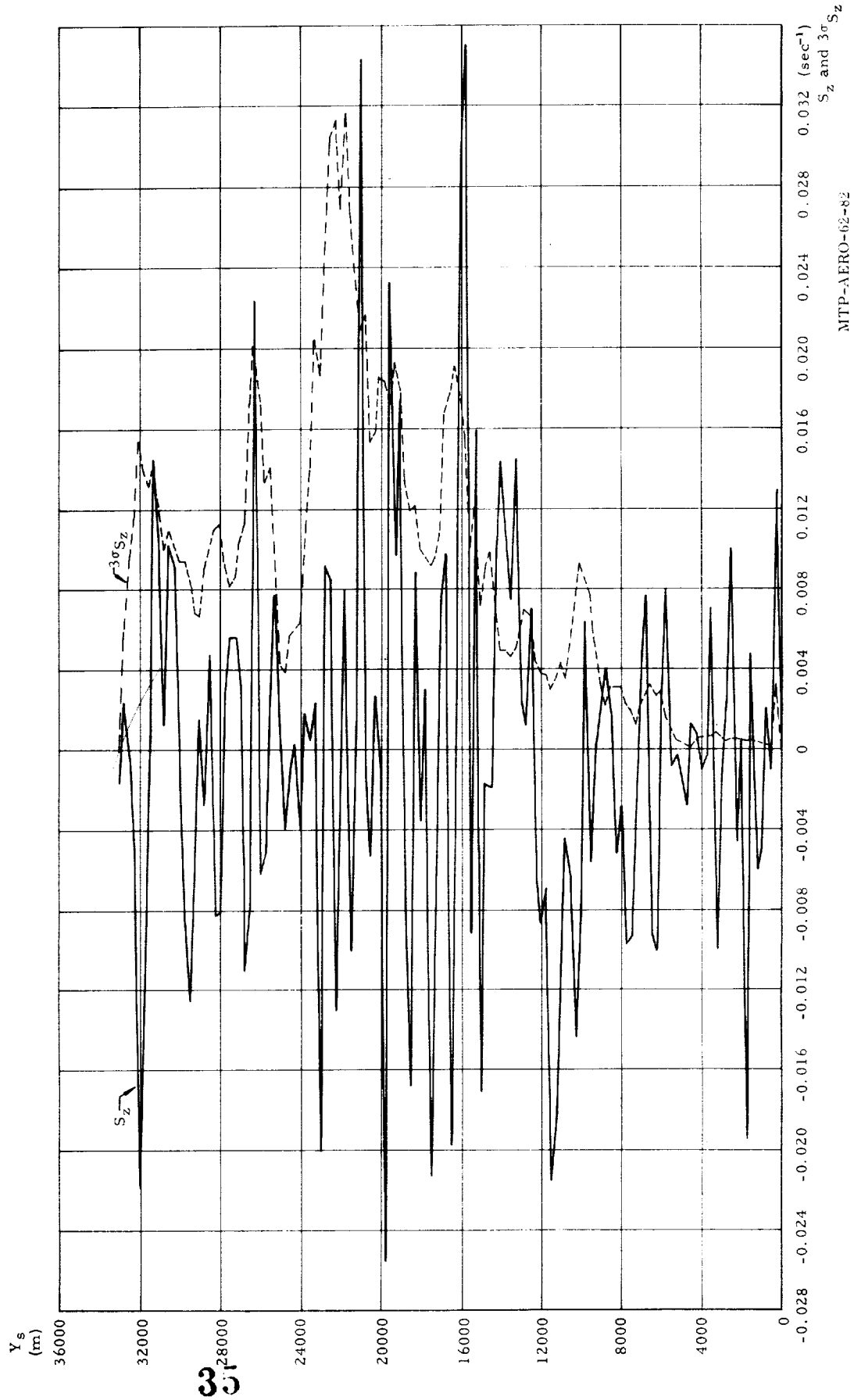


Fig. 9. Height Sequenced Crossrange Component Wind Shear and  $3\sigma$  Error in Crossrange Component Wind Shear over 250 m Layers, Cape Canaveral, Florida, October 27, 1961, 1513Z

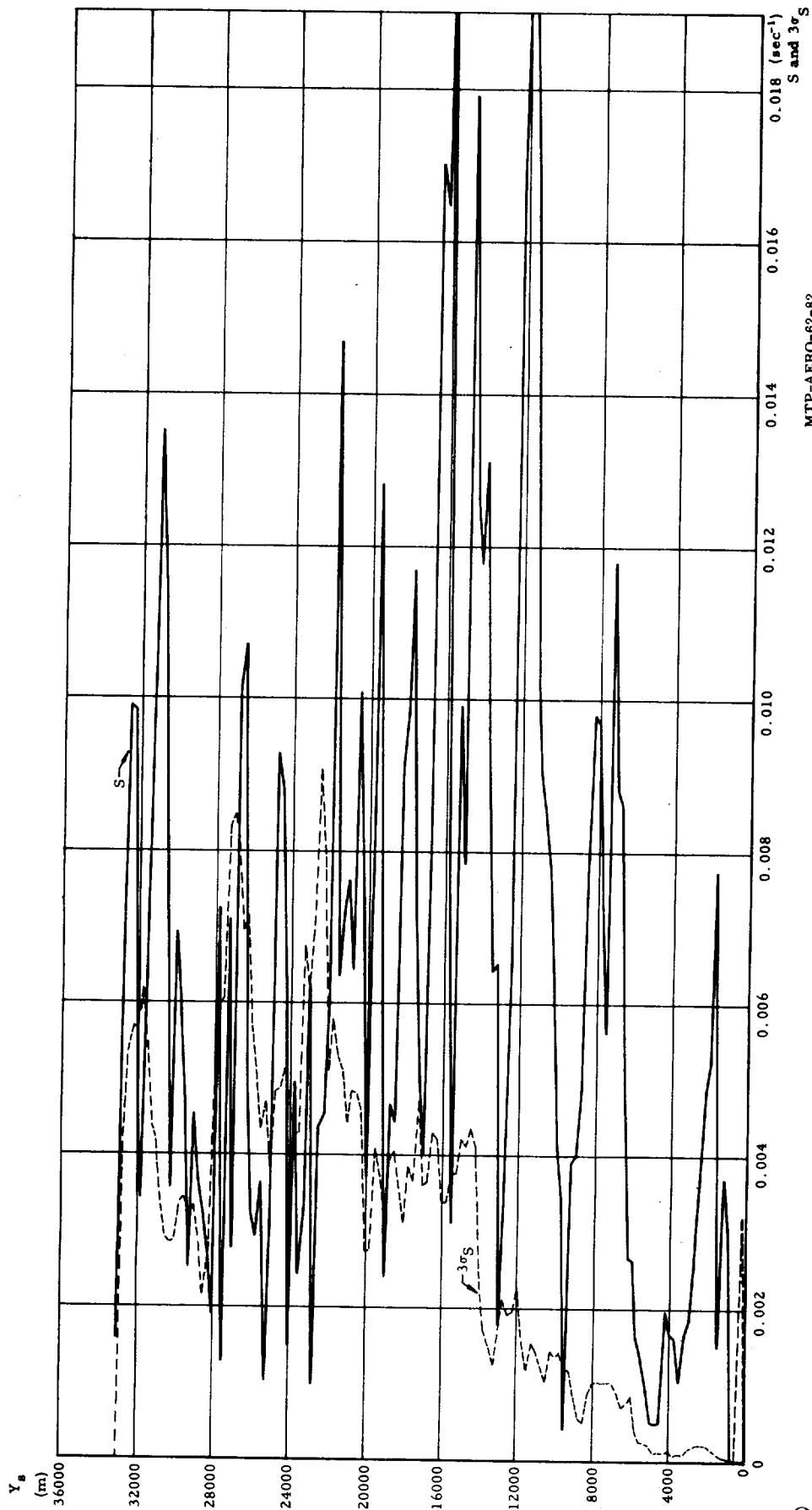


Fig. 10. Height Sequenced Wind Shear and 3σ Error in Wind Shear over 1000 m Layers,  
Cape Canaveral, Florida, October 27, 1961, 1513Z

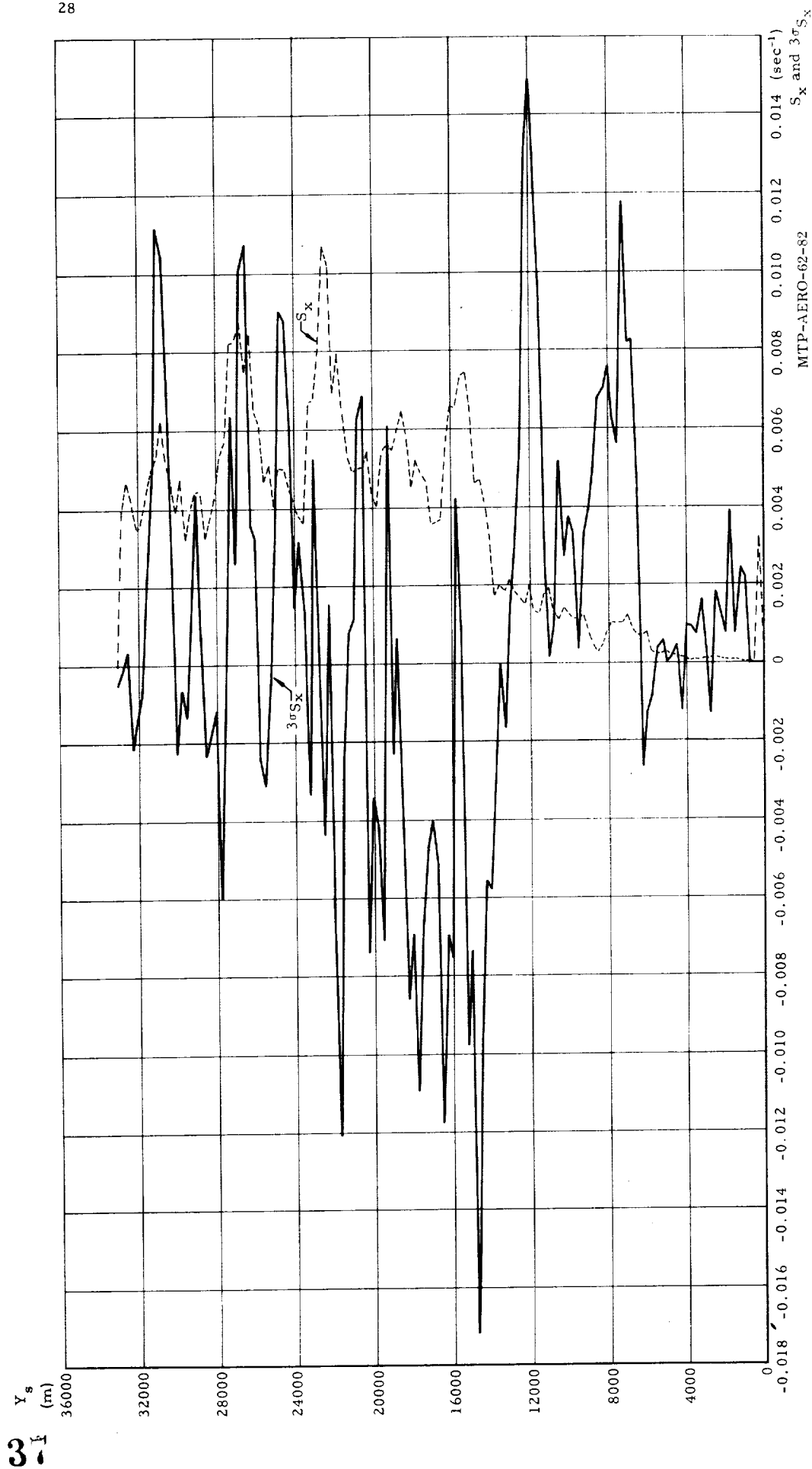


Fig. 11. Height Sequenced Range Component Wind Shear and  $3\sigma$  Error in Range Component  
Wind Shear over 1000 m Layers, Cape Canaveral, Florida, October 27, 1961, 1513Z



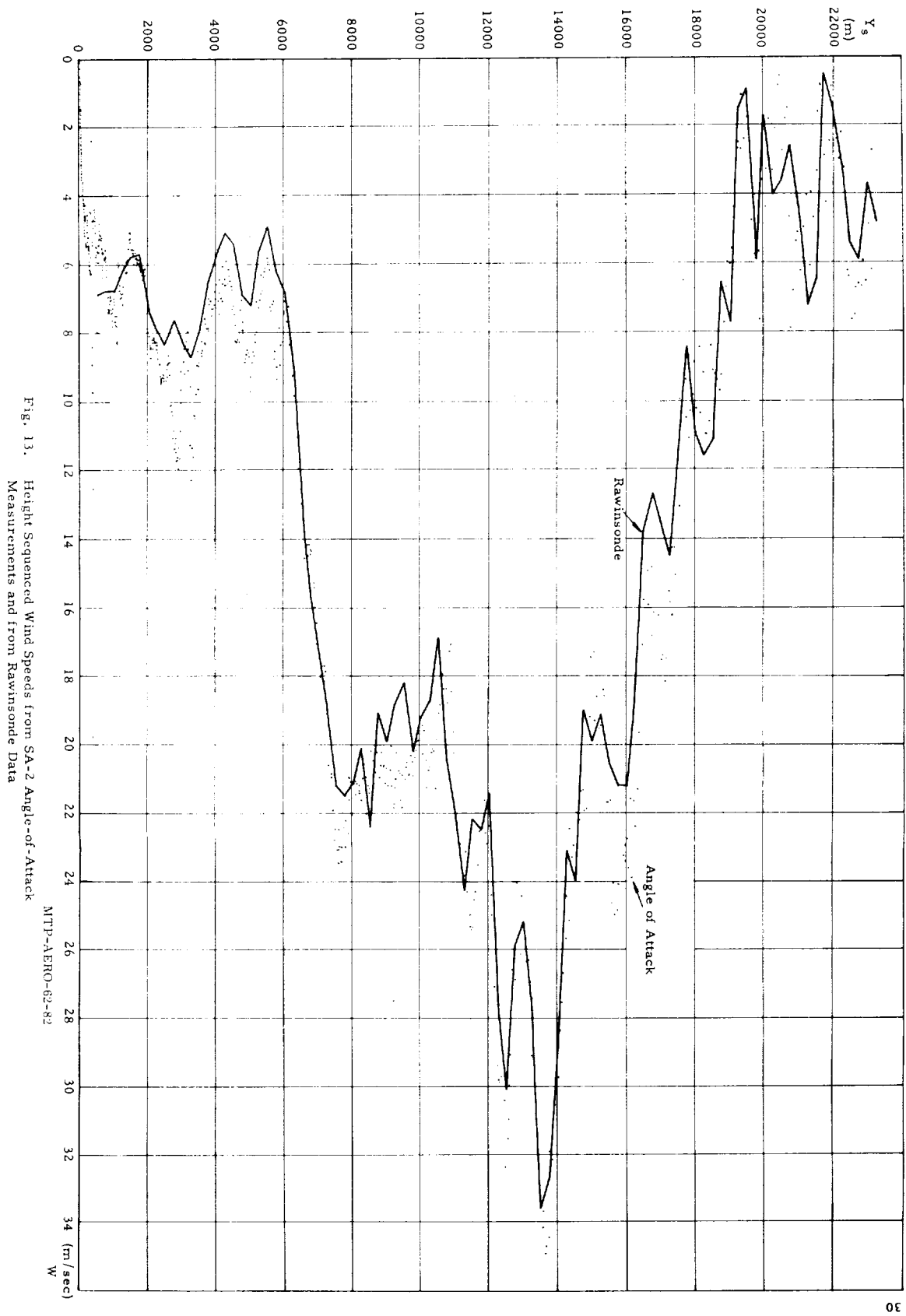


Fig. 13. Height Sequenced Wind Speeds from SA-2 Angle-of-Attack Measurements and from Rawinsonde Data

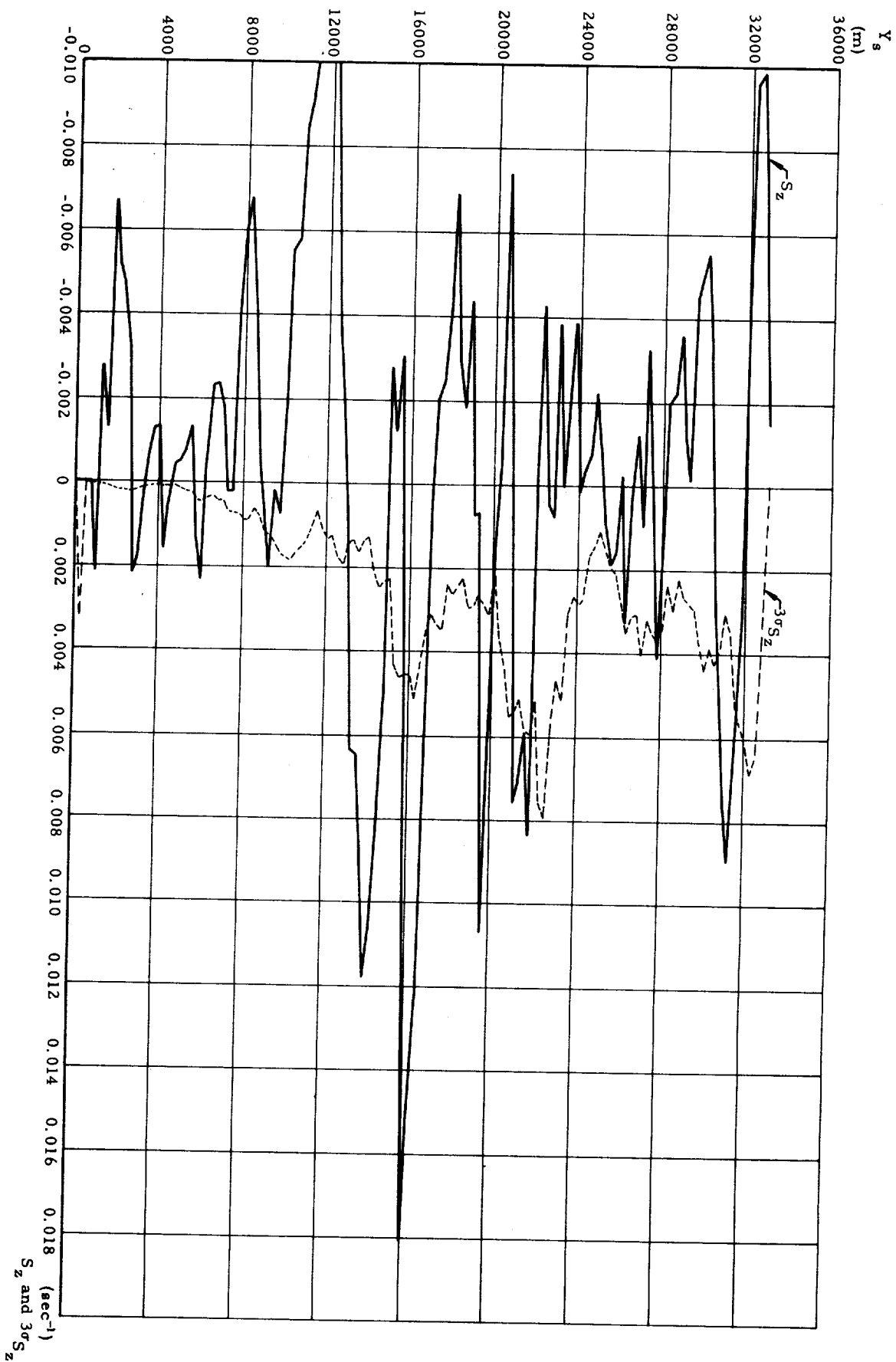


Fig. 12. Height Sequenced Crossrange Component Wind Shear and  $3\sigma$  Error in Crossrange Component Wind Shear over 1000 m Layers, Cape Canaveral, Florida, October 27, 1961, 1513Z

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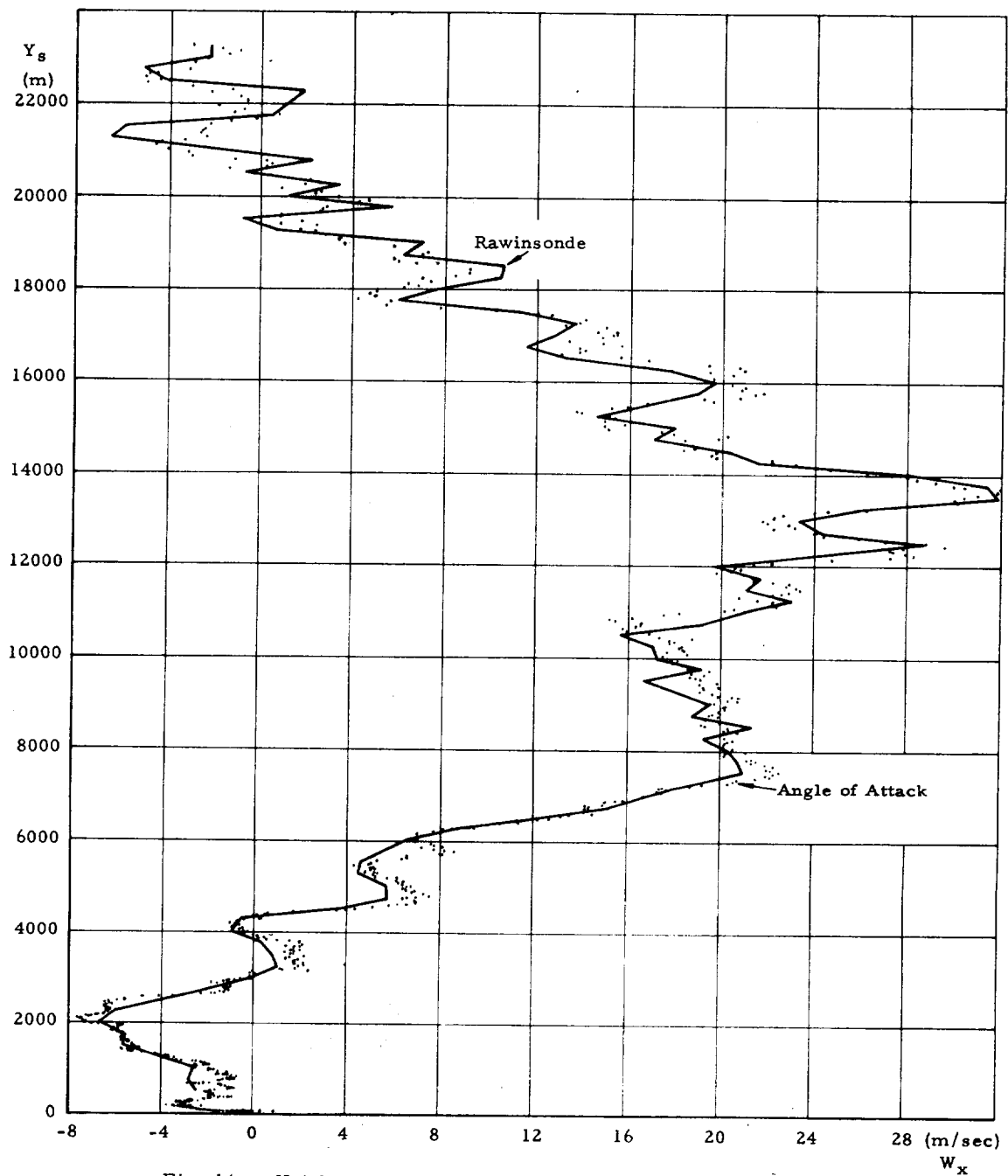
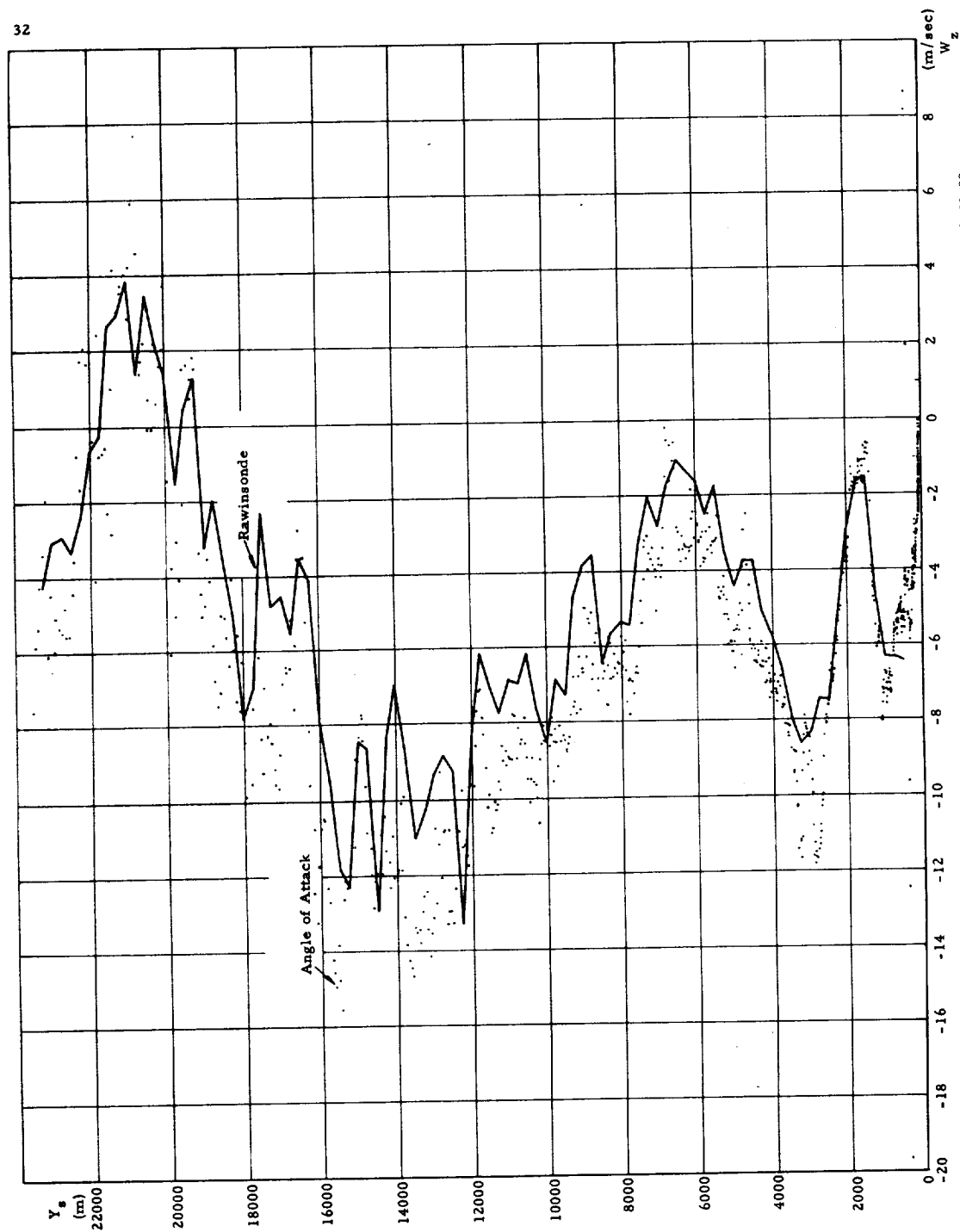


Fig. 14. Height Sequenced Range Component Wind Speeds from SA-2 Angle-of-Attack Measurements and from Rawinsonde Data MTP-AERO-62-82




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Fig. 15. Height Sequenced Crossrange Component Wind Speeds from SA-2 Angle-of-Attack Measurements and from Rawinsonde Data

## APPROVAL

ROOT-MEAN-SQUARE ERROR ANALYSIS FOR EQUATIONS IN  
RAWINSONDE EVALUATION PROGRAM

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.




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